SHENTON COLLEGE

ATMAM Mathematics Methods

Test 1 2018 Calculator Free

Name: Solutions

Teacher: Friday Smith

Time Allowed : 30 minutes

Marks	/31

Materials allowed: Formula Sheet.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given as exact values.
Marks may not be awarded for untidy or poorly arranged work.

1. [2,2,2,2] 8

Differentiate each of the following with respect to x, clearly showing appropriate rules Do not simplify answers.

(a) $y = \frac{1}{2}x^3 - \frac{2}{x^2} + 5$

$\frac{dy}{dx} = \frac{3}{2}x^2 + \frac{4}{x^3}$

✓ polynomial term.

✓ reciprocal term

(b) $y = \frac{\cos x}{x^4+2}$

$\frac{dy}{dx} = \frac{(x^4+2)(-\sin x) - \cos x\,(4x^3)}{(x^4+2)^2}$

✓ quotient rule demonstrated

✓ $\frac{d}{dx}\cos x = -\sin x$

(c) $y = \sqrt{3x^2+4}$

$\frac{dy}{dx} = \frac{1}{2}(3x^2+4)^{-\frac{1}{2}}\cdot(6x)$

✓ $\frac{d}{dx}\sqrt{}$

✓ $\frac{d}{dx}\ 3x^2+4$

(d) $y = e^{-x}\sin x$

$\frac{dy}{dx} = e^{-x}\cos x + \sin x\,(e^{-x})(-1)$

✓ use of product rule

✓ use of chain rule for e^{-x}.

2. [1,1] 2

Evaluate each of the following limits.

(a) $\lim_{h\to 0}\frac{e^h-1}{h}$

$= 1$

✓ evaluates limit

(b) $\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)$

$= -\sin x$

✓ evaluates limit

3. [3.1] 4

Determine the value of $f''(-1)$ if $f(x) = (2x+1)^5$.

Describe the concavity of the curve at this point. and explain.

$f(x) = (2x+1)^5$

$f'(x) = 10(2x+1)^4$ ✓ $f'(x)$

$f''(x) = 80(2x+1)^3$ ✓ $f''(x)$

$f''(-1) = -80$ ✓ $f''(-1)$

Concave down as gradient function is decreasing ✓

OR

$f''(x) < 0$ would apply to a maximum T.P. wich is concave down

✓ correct concavity with reason

4 [4]

Find the equation of the tangent to $y = 3 - \sin(1 - 2x)$ at the point where $x = \frac{1}{2}$.

$$\frac{dy}{dx} = -\cos(1-2x)(-2)$$

$$= 2\cos(1-2x)$$

✓ $\frac{dy}{dx}$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}} = 2\cos 0$$

$$= 2$$

✓ $\frac{dy}{dx}$ $x = \frac{1}{2}$

Equation of tangent

$$y = 2x + c$$

✓ recognize $m = 2$ from $\frac{dy}{dx}$.

when $x = \frac{1}{2}$ $y = 3 - \sin 0$ $\left(\frac{1}{2}, 3\right)$

$$3 = 2\left(\tfrac{1}{2}\right) + c$$

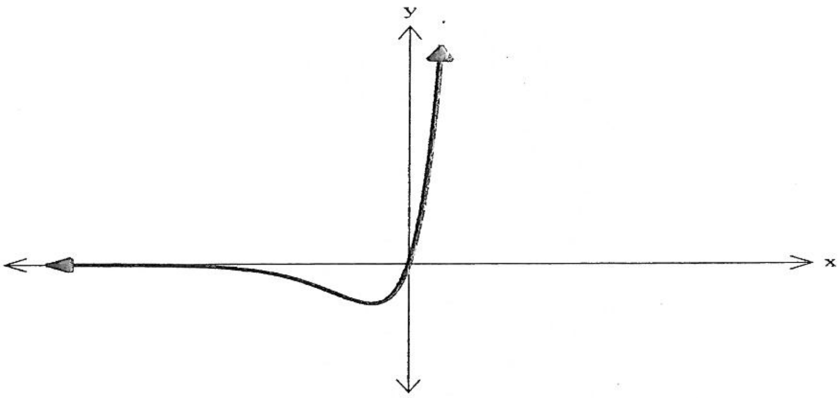
$$c = 2$$

$$y = 2x + 2$$

✓ equation

5. [3,2,3] (8)

The graph of $y = f(x)$ is shown below, where $f(x) = 2xe^x$



(a) Determine the exact location of the stationary point on the graph of $y = f(x)$.

$f(x) = 2xe^x$ ✓ $f'(x)$

$f'(x) = 2x \cdot e^x + e^x \cdot (2)$ ✓ $f'(x) = 0$ determined

$= 2e^x(x+1)$

$f'(x) = 0$ when $x = -1$ $\quad e^x > 0$ for all x ✓ exact co-ord

$f(-1) = 2(-1)e^{-1}$

$\therefore$ Stationary point at $\left(-1, \frac{-2}{e}\right)$

(b) Apply the second derivative test to show that the stationary point in (a) is a minimum.

$f''(x) = 2e^x(1) + (x+1)2e^x$ $\quad \frac{2}{e}$ (1)

$= 2e^x(2+x)$ $\quad 4e^x + 2xe^x$

$f''(-1) > 0 \quad \therefore$ minimum stationary point

✓ $f''(x)$

✓ apply test correctly.

(c) The graph of $y = f(x)$ has just one point of inflection. Determine the exact coordinates of this point.

$f''(x) = 0$ for point of inflection ✓ $f''(x) = 0$

✓ check concavity change

$4e^x + 2xe^x$ $\quad 2e^x(2+x) = 0$

$2e^x > 0$ for all x $\therefore x = -2$

check change of concavity

x	-3	-2	-1
$f''(x)$	$-$	0	$+$

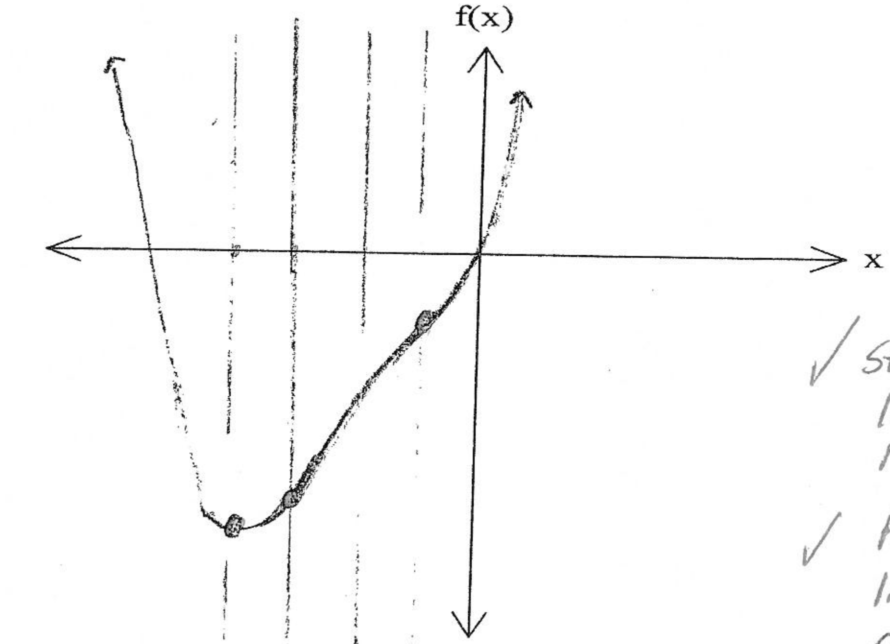
✓ exact co-ord.

$\therefore$ Point of Inflection at $\left(-2, \frac{-4}{e^2}\right)$

6 [5]

Given the graph of $y = f'(x)$ provide possible graphs of $y = f(x)$ and $y = f''(x)$

[Care should be taken with the x values of critical points, but the 'heights' of the derivatives are not unique, use whatever makes your sketch easier to draw.]

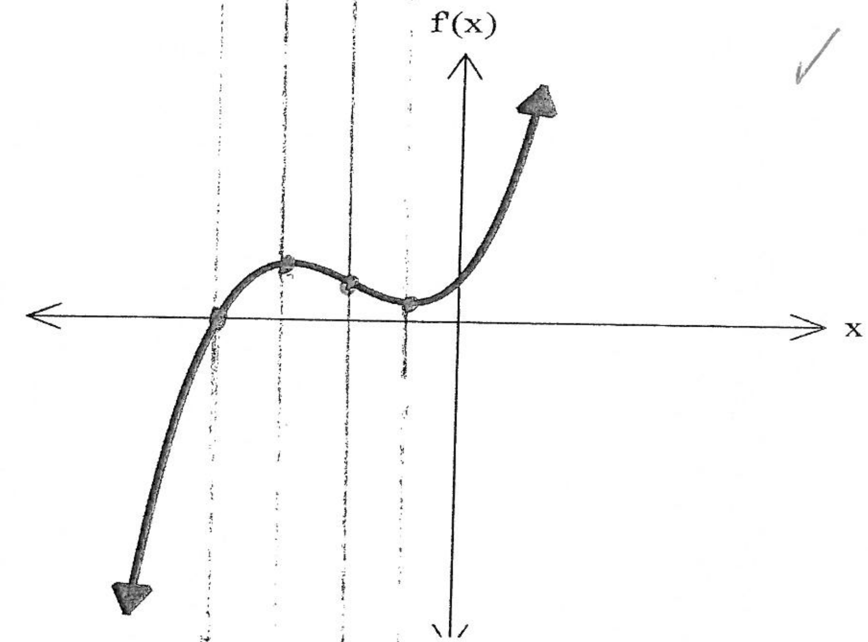


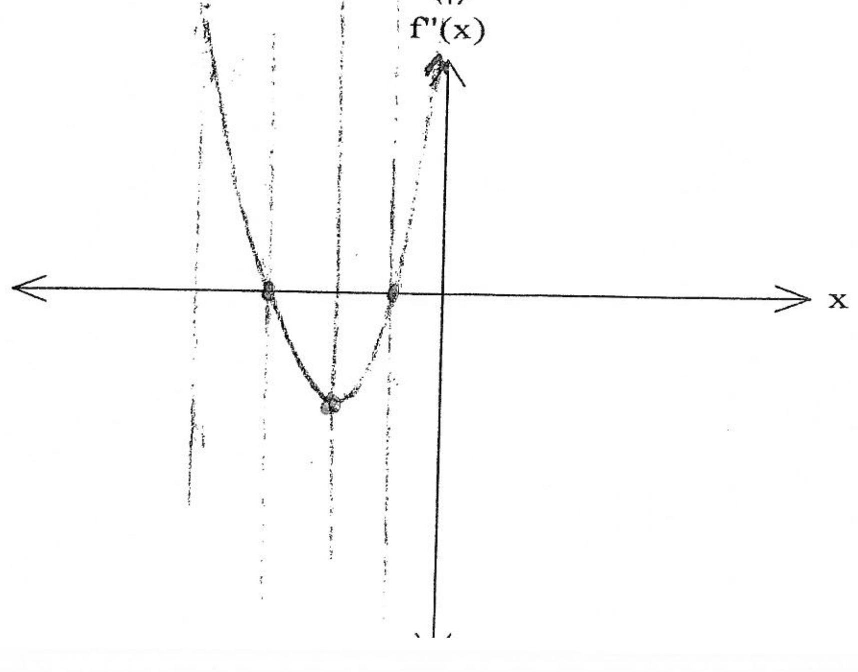
✓ Stationary point on f(x) lines up with root of f'(x)

✓ Points of Inflection on f(x line up with change in gradient on f'(x)

✓ Gradients correct

− + ++





✓ f''(x) = 0 lines up with inflection point on f (x)

✓ correct f''(x) graph

ATMAM Mathematics Methods

Test 1 2018

Calculator Assumed

SHENTON
COLLEGE

Name: ....SOLUTIONS...

Teacher: Friday Smith

Time Allowed : 20 minutes

Marks	/19

Materials allowed: Classpad, calculator, formula sheet.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given to <u>two</u> decimal places.
Marks may not be awarded for untidy or poorly arranged work.

7. **[1,1,1,1]** 4

The number of bees in a hive after t months is modelled by $B(t) = \frac{3000}{1+0.5e^{-1.73t}}$.

Determine:

(a) Determine the initial bee population.

2000 bees

✓ $B(0)$

(b) Determine the percentage increase in its population after one month.

37.79%

✓ correct % increase

(c) Explain why the population is increasing over time.

$B'(t) > 0$ for all values of t.

✓ suitable explanation should mention $B'(t)$

(d) Determine the rate at which the population is increasing after 3 months.

$B'(3) = 14.38$ bees/month.

✓ $B'(3)$ with rate unit

8. [1,1,1,2,4] 9

On the Indonesian coast, the depth of water t hours after midnight s given by $D(t) = 9.3 + 6.8\cos(0.507t)$ metres $0 \le t \le 24$

(a) Find the depth of the water at 8 am.

5·15 m

✓ correct depth

(b) Determine the maximum height of the water during this time.

16·10 m

✓ max depth

(c) At what rate is the water changing at 8 am?

2·73 m/h

✓ correct rate of change

(d) At what time of day is water rising at its fastest rate?

9·29 h   21·69 h

9·17 am   9·41 pm

OR 9·18 am

✓✓ correct times one each

only ① if hours only

(e) Show **how** to use calculus to determine the time(s) of day the height increasing at 1.5 metres per hour. Use your calculator to help you determine the time(s).

$D'(t) = -3{\cdot}4476 \sin(0{\cdot}507t)$

Require $D'(t) = 1{\cdot}5$

$t = 7{\cdot}08,\ 11{\cdot}51,\ 19{\cdot}48$ and $23{\cdot}90$ h

7:05 am, 11:30 am, 7:27 pm and 11:54 pm

Penalise ONCE

−1 no 2dp not 16·10 m

−1 rounding/accuracy

−1 rate unit

−1 units.

✓ differentiate

✓ = 1·5

✓ solve for t

✓ correct times all

9. **[1,1,1,1,1,1]** 6

The population of a city over t years is given by $P = 120\ 000e^{0.07t}$

(a) Determine the population after 10 years.

241 650 ✓ correct population

(b) Find how long it takes for the population to double in size.

9.9 years ✓ correct no of years.

(c) Express the rate of growth as a function of t.

$\frac{dP}{dt} = 8400e^{0.07t}$ ✓ $\frac{dP}{dt}$

(d) Determine the rate of growth after 10 years.

$\approx$ 16 916 persons/year ✓ correct rate.

(e) Express the rate of growth as a function of P

$\frac{dP}{dt} = 0.07P$ ✓ correct function of P

(f) Determine the growth rate when the Population is 3 million.

$0.07 \times 3\ 000\ 000$

$\approx 210\ 000$ ✓ growth rate

End of Questions